



Trinity College

WA Exams Practice Paper E, 2016

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNIT 3

Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

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|--|--|--|--|--|--|--|--|
| | | | | | | | |
|--|--|--|--|--|--|--|--|

In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| Total | | | | 150 | 100 |

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free

35% (52 Marks)

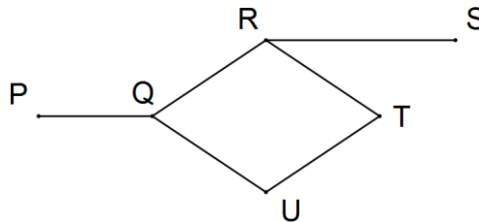
This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1

(5 marks)

A simple graph is shown below.



(a) Explain why the graph is simple.

(2 marks)

The graph contains

- no loops
- no multiple edges

(b) Clearly explain why the graph contains a Hamiltonian path but not a Hamiltonian cycle.

(3 marks)

Hamiltonian paths are open paths that visit every vertex once only. For example, PQUTRS.

A Hamiltonian cycle does not exist as they must be closed paths that visit every vertex once only, except for start/end vertex.

Because PQ and RS are both bridges, a closed path/cycle cannot exist.

*(Bridges **must** be travelled twice as part of a closed path, which contradicts the definition of a path in which all edges must be travelled no more than once).*

Question 2

(8 marks)

(a) Types of data can be described using the words categorical, numerical, nominal, ordinal, discrete and continuous. Choose two of these words that best describe each of the following data types:

(i) The number of students who sit an exam. (1 mark)

Numerical, discrete.

(ii) The ranking of students results in an exam. (1 mark)

Categorical, ordinal.

(b) A class was set a task to investigate whether an association exists between a student's height and their hand span.

(i) What type of graph would be appropriate to analyse data collected? (1 mark)

Scatterplot

(ii) What statistical measure would be appropriate to calculate as part of the data analysis? (1 mark)

Correlation coefficient

(iii) One student designed the questionnaire shown below. Comment on the appropriateness of their design. (2 marks)

| | | | | | | | |
|---------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Height in cm (circle one) | 150- 155 | 155- 160 | 160- 165 | 165- 170 | 170- 175 | 175- 180 | 180- 185 |
| Hand span in cm (circle one) | 16 | 17 | 18 | 19 | 20 | 21 | 22 |

Not very appropriate or useful
 - Better to record exact measurements for each person rather than group
 - Height group boundaries are ambiguous
 - Height and hand groups do not allow for measurements outside given groups

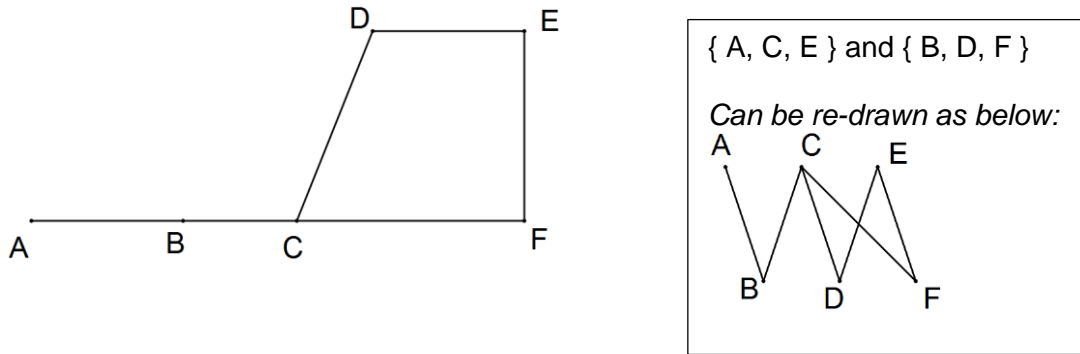
(iv) A student carried out the investigation, found that a strong positive association existed, and concluded that having a large hand span was due to being tall. Comment on their conclusion. (2 marks)

Conclusion implies that one thing causes another, which may not necessarily be true. All that can be concluded is that an association exists between the variables.

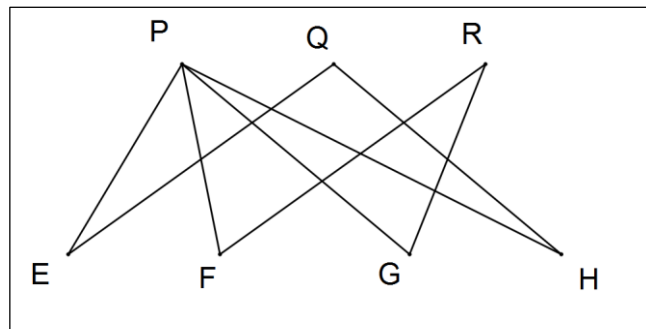
Question 3

(7 marks)

- (a) Two sets of points are shown in the bipartite graph below. List the vertices belonging to each of the sets. (2 marks)



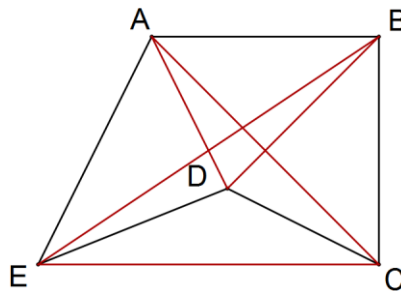
- (b) Three songs, labelled P, Q and R, were released over a number of years on a variety of compilation CDs, labelled E, F, G and H, as follows: P was released on all four CDs; Q was released on E and H; and R was released on F and G. Show this information as a bipartite graph. (3 marks)



- (c) State whether the graph with five vertices shown below is complete. If it is, explain why. If not, add any missing edges. (2 marks)

Not complete.

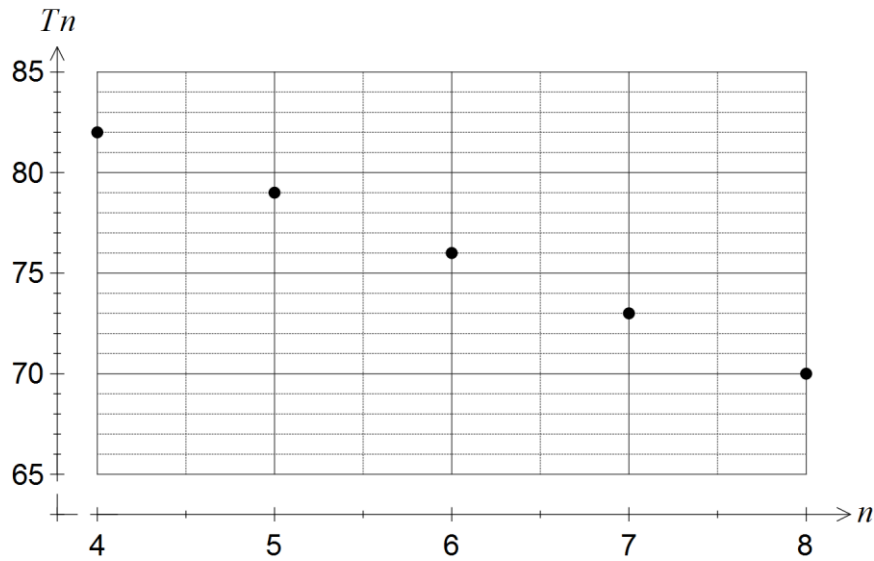
(All vertices must connect to all other vertices.)



Question 4

(7 marks)

The terms of a sequence are shown in the graph below.



- (a) Describe the feature of the graph that indicates the sequence is arithmetic. (1 mark)

Points lie in a straight line.

- (b) Determine

(i) T_9 .

(1 mark)

$$T_9 = 70 - 3 = 67$$

(ii) T_1 .

(1 mark)

$$T_1 = 82 + 3 \times 3 = 91$$

- (c) The rule for the n^{th} term of the sequence is $T_n = an + b$. Determine the values of the constants a and b . (2 marks)

$$\begin{aligned} T_n &= 91 + (n - 1) \times (-3) \\ &= 91 - 3n + 3 \\ &= -3n + 94 \Rightarrow a = -3, b = 94 \end{aligned}$$

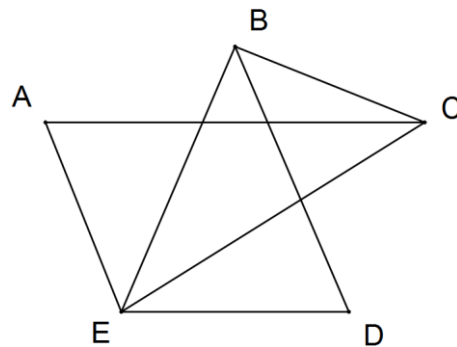
- (d) Determine the smallest value of n such that $T_n < 0$. (2 marks)

$$\begin{aligned} 94 - 3n &= 0 \\ 3n &= 94 \\ n &= 31\frac{1}{3} \\ \text{Smallest value of } n &\text{ is } 32. \end{aligned}$$

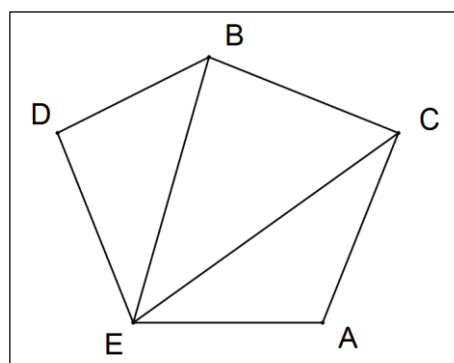
Question 5

(9 marks)

A graph with five vertices is shown below.



- (a) Redraw the graph with no edges crossing to show that it is planar. (2 marks)



- (b) State the number of edges and faces and hence show that the graph satisfies Euler's formula. (3 marks)

7 edges and 4 faces.
 $5 + 4 = 7 + 2$
 $9 = 9 \Rightarrow$ satisfies Euler's formula

- (c) List, in order, the vertices of a trail that shows the graph is semi-Eulerian. (2 marks)

B D E B C E A C
(Others exist but trail must include every edge once only and will start and finish at B and C or vice versa)

- (d) An edge can be added to the graph so that the graph becomes Eulerian.

- (i) Between which two vertices should the edge be added? (1 mark)

B and C.

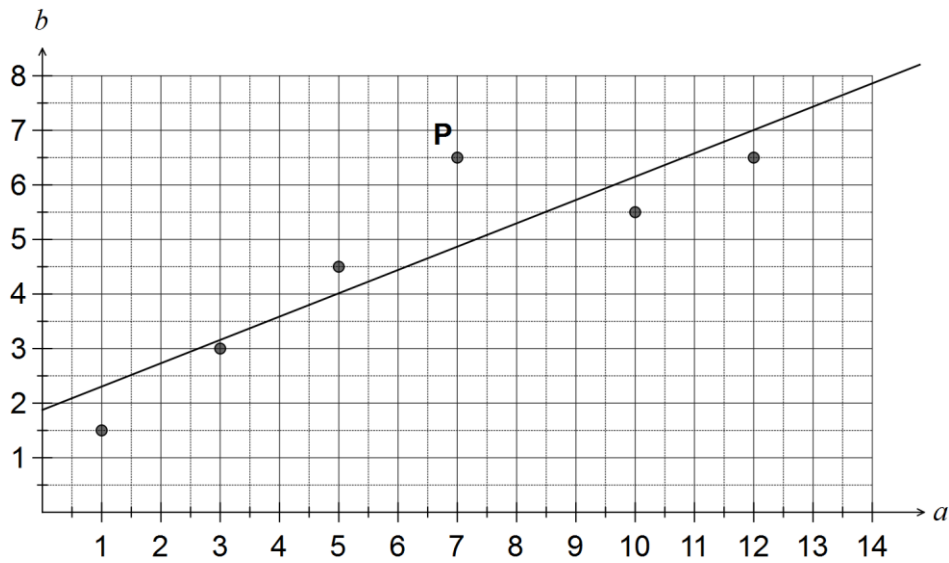
- (ii) If a suitable edge is added, how many of the vertices are suitable for the start of a Eulerian trail? (1 mark)

All 5 of the vertices.

Question 6

(8 marks)

The scatterplot and least-squares line for a set of bivariate data (a, b) with correlation coefficient 0.9 is shown.



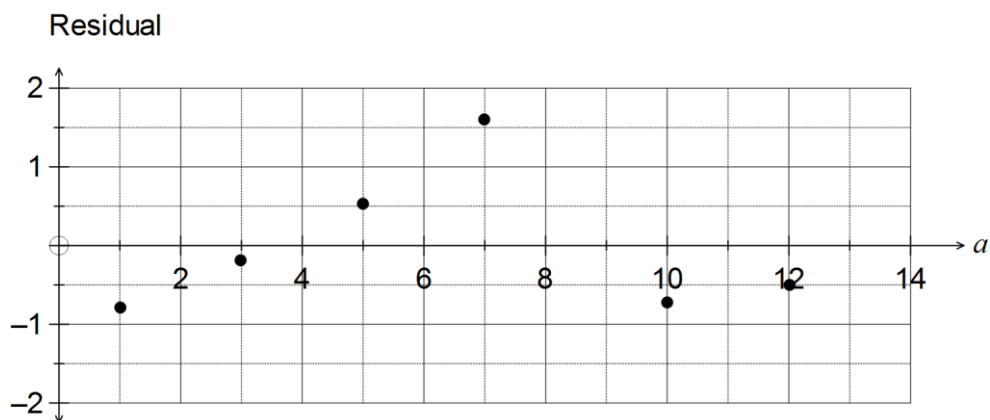
- (a) Describe the effect on the correlation coefficient if the point labelled P was removed from the dataset. (1 mark)

Coefficient would increase, becoming closer to 1.

- (b) Describe the effect on the least-squares line if all data points with $a > 8$ were removed from the dataset. (2 marks)

- gradient would increase.
- vertical-axis intercept would decrease.

- (c) Construct a residual plot for the six paired values on the axes below. (3 marks)



- (d) Comment, with reasons, on the appropriateness of fitting a linear model to this dataset. (2 marks)

Not appropriate because a pattern is evident with the points in the residual plot.

Question 7

(8 marks)

(a) Given that $T_{n+1} = \frac{1}{2}T_n + 4$, $T_3 = 14$, determine

(i) T_4 .

(1 mark)

$$T_4 = \frac{1}{2} \times 14 + 4 = 11$$

(ii) T_1 .

(2 marks)

$$14 = \frac{1}{2}T_2 + 4 \Rightarrow T_2 = 20$$

$$20 = \frac{1}{2}T_1 + 4 \Rightarrow T_1 = 32$$

(iii) The long term steady-state solution of the sequence.

(2 marks)

In long term, $T_{n+1} = T_n = x$.

$$x = \frac{1}{2}x + 4$$

$$\frac{1}{2}x = 4$$

$$x = 8$$

(b) The first three terms of a sequence given by $T_{n+1} = aT_n + b$, $T_1 = 2$ are 2, 5 and 17.

Determine the values of the constants a and b .

(3 marks)

$$5 = 2a + b$$

$$17 = 5a + b$$

$$3a = 12 \Rightarrow a = 4$$

$$5 = 2(4) + b \Rightarrow b = -3$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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